B.Sc. (Part-II) (CBCS Pattern) Semester-IV

USMT-07 - Mathematics Paper-I - Algebra

P. Pages : 2 **GUG/S/25/12014**

Time : Three Hours Max. Marks : 60

* 1 8 2 0 *

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) If G is a group such that $(ab)^2 = a^2b^2, \forall a, b \in G$. Show that G must be abelian. **6**

b) Prove that the union of two subgroups is not necessarily a subgroup. **6**

OR

c) Let a be any element of a multiplicative group G. Then prove that $0(a) = 0\left(a^{-1}\right)$. **6**

d) For $s = \{1,2,3,4,5,6,7,8,9\}$ and $a, b \in A(s)$, compute $a^{-1}ba$, where $a = (1,3,5)(1, 2)$, $b = (1,5\ 7\ 9)$. **6**

UNIT – II

2. a) Prove that a subgroup N of G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G. **6**

b) If H is a subgroup of G and N is normal subgroup of G, show that $H \bigcap N$ is a normal subgroup of H. **6**

OR

c) Let G be any group, g is a fixed element in G. Define $\phi : G \to G$ by $\phi(x) = gxg^{-1}$. Prove that ϕ is an isomorphism of G onto G. **6**

d) Let G be the group of non-zero real numbers under multiplication and $G' = \{1, -1\}$ where $1 \cdot 1 = 1,\ 1 \cdot (-1) = (-1) -1 = -1, (-1)(-1) = 1$. Define $\phi : G \to G'$ such that $\phi(x) = \begin{cases} 1 & x \text{ is positive} \\ -1 & x \text{ is negative} \end{cases}$ show that ϕ is a homomorphism. **6**

UNIT – III

3. a) If ϕ is an homomorphism of a group G in to a group G'. Then prove that **6**

i) $\phi(e) = e'$ and ii) $\phi\left(x^{-1}\right) = \left(\phi(x)\right)^{-1}, \forall x \in G.$ where e and e' are the identities of G and G' respectively.

b) If ϕ is a homomorphism of G into G' with kernel k, then prove that k is a normal subgroup of G. **6**

OR

c) Prove that any infinite cyclic group is isomorphic to the additive group of integers. **6**

d) If M, N are normal subgroups of G, prove that $\frac{NM}{M} \approx \frac{N}{N \cap M}$. **6**

UNIT – IV

4. a) Prove that a ring R is commutative iff $(a+b)^2 = a^2 + 2ab + b^2$. **6**

b) If R is a ring with zero element 0, then prove that **6**
i) $a0 = 0a = 0$ and ii) $a(-b) = (-a)b = -(ab),\ \forall a, b, c \in R$

OR

c) Prove that the intersection of two subring is a subring. **6**

d) Prove that a ring R is without zero divisor if and only if the cancellation laws holds in R. **6**

5. Solve **any six.**

a) If G is a group such that $(ab)^2 = a^2b^2,\ \forall a, b \in G,$ show that G must be abelian. **2**

b) Prove that an identity of a group G is unique. **2**

c) Write definition of normal subgroup. **2**

d) If G is a finite group and N is a normal subgroup of G, then prove that $0(G/n) = 0(G)/0(N)$. **2**

e) Write definition of Homomorphism. **2**

f) Prove that any kernel is non-empty. **2**

g) Write down definition of ring. **2**

h) Write down definition of Ring with no zero divisor. **2**

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